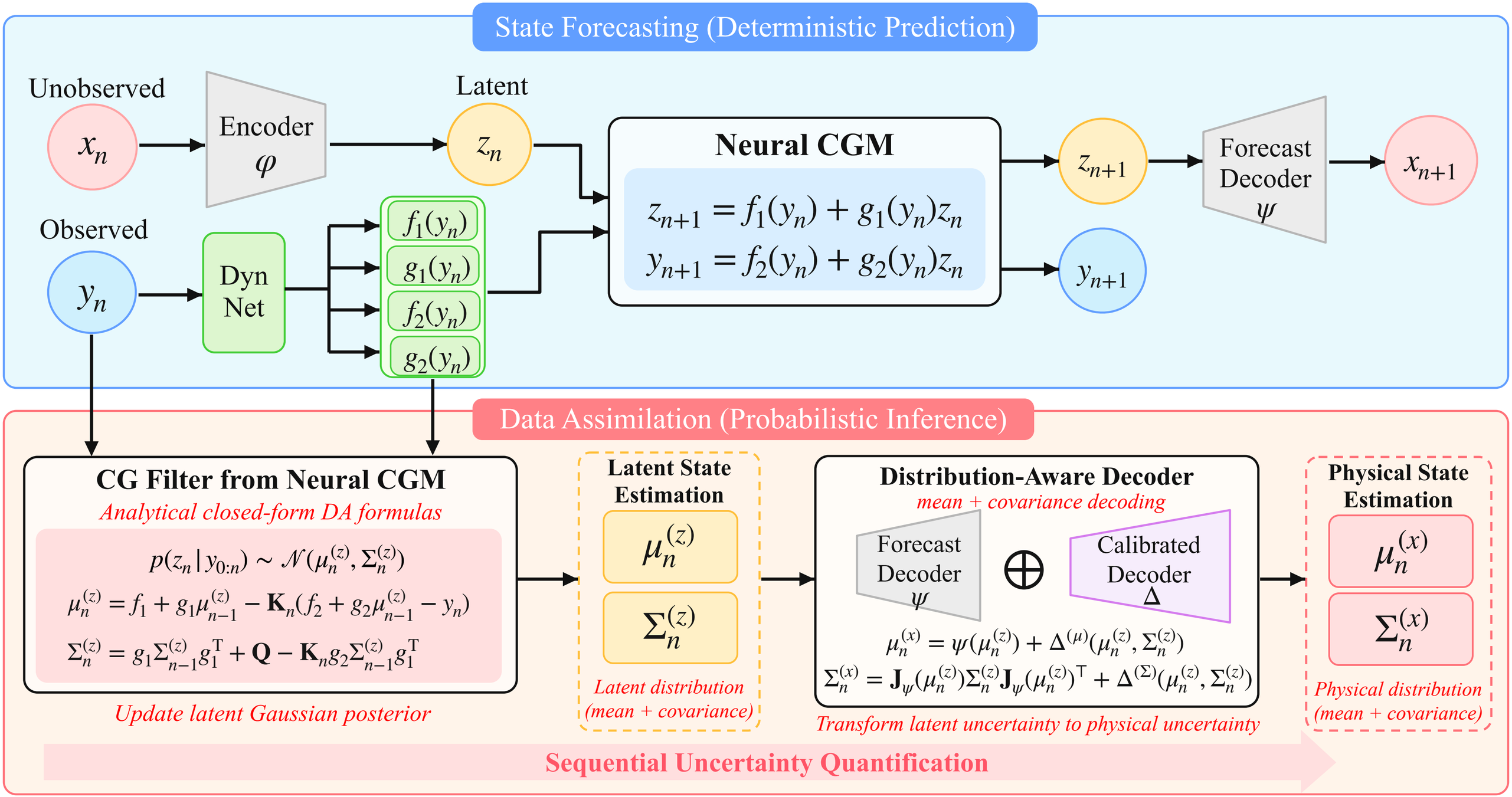

State Forecasting (Deterministic Prediction)
Unobserved
x_n
Encoder
φ
Latent
z_n
Observed
y_n
Dyn Net
$f_1(y_n)$
$g_1(y_n)$
$f_2(y_n)$
$g_2(y_n)$
Neural CGM
$z_{n+1} = f_1(y_n) + g_1(y_n) z_n$
$y_{n+1} = f_2(y_n) + g_2(y_n) z_n$
z_{n+1}
y_{n+1}
Forecast Decoder
ψ
x_{n+1}
Data Assimilation (Probabilistic Inference)
CG Filter from Neural CGM
Analytical closed-form DA formulas
$p(z_n \mid y_{0:n}) \sim \mathcal{N}(\mu_n^{(z)}, \Sigma_n^{(z)})$
$\mu_n^{(z)} = f_1 + g_1 \mu_{n-1}^{(z)} - \mathbf{K}_n (f_2 + g_2 \mu_{n-1}^{(z)} - y_n)$
$\Sigma_n^{(z)} = g_1 \Sigma_{n-1}^{(z)} g_1^{\mathrm{T}} + \mathbf{Q} - \mathbf{K}_n g_2 \Sigma_{n-1}^{(z)} g_1^{\mathrm{T}}$
Update latent Gaussian posterior
Latent State Estimation
$\mu_n^{(z)}$
$\Sigma_n^{(z)}$
Latent distribution (mean + covariance)
Distribution-Aware Decoder
mean + covariance decoding
Forecast Decoder
ψ
Calibrated Decoder
Δ
$\mu_n^{(x)} = \psi(\mu_n^{(z)}) + \Delta^{(\mu)}(\mu_n^{(z)}, \Sigma_n^{(z)})$
$\Sigma_n^{(x)} = \mathbf{J}_\psi(\mu_n^{(z)}) \Sigma_n^{(z)} \mathbf{J}_\psi(\mu_n^{(z)})^{\top} + \Delta^{(\Sigma)}(\mu_n^{(z)}, \Sigma_n^{(z)})$
Transform latent uncertainty to physical uncertainty
Physical State Estimation
$\mu_n^{(x)}$
$\Sigma_n^{(x)}$
Physical distribution (mean + covariance)
Sequential Uncertainty Quantification