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SEMIPROJECTIVITY OF UNIVERSAL C*-ALGEBRAS GENERATED BY ALGEBRAIC ELEMENTS

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Abstract. Let p be a polynomial in one variable. It is shown that the universal C^*-algebra of a relation $p(x) = 0$ is semiprojective and residually finite-dimensional. Applications to polynomially compact operators are given.

Introduction

The "general lifting problem" in C*-algebras is a class of problems which are formulated in the following way: for a quotient C^*-algebra A/I and its element b with some specific properties, to find (or to prove the existence of) a preimage of b in A with the same properties. Lifting problems are closely related to the theory of stability of relations which, roughly speaking, investigates when "an almost representation of a relation" is "close" to a representation (for strict definitions see [6]).

By a relation we mean a system of equalities of the form $f(x_1, ..., x_n, x_1^*, ..., x_n^*) = 0$, where f is a non-commutative polynomial (an element of the corresponding free algebra), which can include also inequalities of the form $\|x_i\| \leq c$ or $\|x_i\| < c$ (they also can be written as polynomial equalities if one extends the list of generators).

Recall that a C^*-algebra D is *projective* if for any C^*-algebra A, its ideal I and every $*$-homomorphism $\phi : D \to A/I$, there exists $\tilde{\phi}$ such that the diagram

$$\begin{array}{ccc} & & A \\ & \overset{\tilde{\phi}}{\nearrow} & \downarrow \\ D & \xrightarrow{\phi} & A/I \end{array}$$

commutes.

A C^*-algebra D is *semiprojective* if for any C^*-algebra A, any increasing chain of ideals I_n in A and for every $*$-homomorphism $\phi : D \to A/I$, where $I = \overline{\bigcup I_n}$, there exist n and $*$-homomorphism $\tilde{\phi} : D \to A/I_n$ such that the diagram

$$\begin{array}{ccc} & & A/I_n \\ & \overset{\tilde{\phi}}{\nearrow} & \downarrow \\ D & \xrightarrow{\phi} & A/I \end{array}$$

commutes.

A C^*-algebra D is *weakly semiprojective* if for any sequence of C^*-algebras A_i and any $*$-homomorphism $\phi : D \to \prod A_i / \bigoplus A_i$, there exists a $*$-homomorphism $\tilde{\phi} : D \to \prod A_i$

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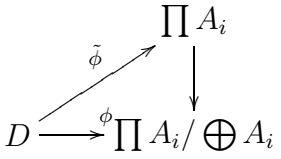
such that the diagram



commutes.

These notions provide an algebraic setting of lifting and stability questions: a relation is liftable (stable, weakly stable) if and only if its universal C^*-algebra is projective (semiprojective, weakly semiprojective), [2], [6]. Of course it is assumed that the universal C^*-algebra of a relation exists. Note that in general a relation needn't have a universal C^*-algebra, but if it includes restrictions $\|x_i\| \leq c_i$, for all generators, then the universal C^*-algebra exists (of course if the relation has at least one representation in a Hilbert space).

Thus given a liftable or (weakly) stable relation there arises a natural question if being combined with restrictions on norms of generators it is still liftable or (weakly) stable: the positive answer would produce new examples of projective or (weakly) semiprojective C^*-algebras.

It was proved in [9] that each nilpotent is liftable and in [6], [7] there was asked a question if a relation

$$x^n = 0, \|x\| \leq 1$$

is liftable or, in other words, if it defines a projective C^*-algebra. In [14] this question was answered positively and the main tool was using the technique of M-ideals in Banach spaces.

In [5] it was proved that for each polynomial p in one variable, a relation $p(x) = 0$ is weakly stable (though the proof given in [5] easily implied also stability) but again it remained unknown if the universal C^*-algebra of a relation

$$p(x) = 0, \|x\| \leq 1$$

is semiprojective. In the present paper we prove this using approach of [14].

Moreover we prove that all these algebras are residually finite-dimensional (have separating families of finite-dimensional representations). Note that this in no way means that these algebras are small. For polynomials of degree more than two they all are non-exact.

Our last result concerns well known questions of C. Olsen. Let $T \in B(H)$, $p(x)$ a polynomial in one variable. Olsen's questions are:

1) Does there exist compact operator K such that $\|p(T+K)\| = \|p(T)\|_e$?
(by $\|T\|_e$ the essential norm of T is denoted).

2) Can this K be chosen common for all polynomials?

The questions are still open but there are partial results ([1], [10], [11], [12], [3], [15]). In particular Olsen proved [10] that in assumption that $p(T)$ is compact, such K always exists. In this paper we prove that in the same assumption K can be chosen common for polynomials $p(x)$ and x. In other words we prove (Theorem 11) that if $p(T)$ is compact then there exists compact operator K such that

$$p(T+K) = 0, \quad \|T+K\| = \|T\|_e.$$

1. Preliminary results and constructions

Below we use the following notation.

By $M(I)$ we denote the multiplier C^*-algebra of a C^*-algebra I.

For a closed ideal I of A, we denote by $a \to \dot{a}$ the standard epimorphism from A to A/I. We say that a is a lift of b if $\dot{a} = b$.

Given ideals $I_1 \subseteq I_2 \subseteq \ldots \subseteq I \subseteq A$ with $I = \overline{\bigcup I_n}$ we use the notation

$$q_n : A \to A/I_n$$

and

$$q_{n,\infty} : A/I_n \to A/I$$

for the induced epimorphisms.

For an arbitrary algebra A, an operator $T : A \to A$ is called an elementary operator if it is of the form $T = \sum_{i=1}^{N} L_{a_i} R_{b_i}$, where $a_i, b_i \in A^+$. By L_{a_i}, R_{b_i} we denote operators of left and right multiplication by a_i and b_i respectively.

A subspace Y of a Banach space X is called proximinal if for any $x \in X$ there is $y \in Y$ such that $\|x-y\| = d(x, Y)$, or, equivalently, if for any $z \in X/Y$ there is a lift x of z such that $\|z\| = \|x\|$.

Theorem 1. *([14]) Let A be a C^*-algebra, T an elementary operator on A. Then, for each ideal I of A, the subspace $\overline{TI}$ is proximinal in $\overline{TA}$.*

This implies a corollary which was contained in nondirect form in the proof of Theorem 5 of [14]:

Corollary 2. *Let A be a C^*-algebra, I its ideal, T an elementary operator on A, $b \in A/I$. If, for some lift a_0 of b, an element Ta_0 is also a lift of b, then the set $\overline{\{Ta|\ \dot{a} = b\}}$ contains an element of norm equal to $\|b\|$.*

Proof. Let $E = \{Ta|\ \dot{a} = b\}$. Then

$$E = \{Ta_0 + Ti \mid\ i \in I\},\ \ \overline{E} = \{Ta_0 + x \mid\ x \in \overline{TI}\}.$$

All elements of $\overline{E}$ are lifts of b. Indeed let $\dot{a} = b$, then $a = a_0 + i$, for some $i \in I$. Since T is elementary, it preserves ideals, whence $(Ti)^{\cdot} = 0$ and $(Ta)^{\cdot} = (Ta_0)^{\cdot} = b$. This easily implies that $\overline{E}$ also consists of lifts of b.

Let $b_i, c_i, i = 1, \ldots, N$, be elements of A^+ such that $A = \sum_{i=1}^{N} L_{b_i} R_{c_i}$ and let B be the C^*-algebra generated by $b_i, i = 1, \ldots, N$, and I. Let us choose a quasicentral approximate unit u_λ for the ideal I of B. Then, for each $a \in E$, we have $T\left((1-u_\lambda)a\right) \in E$ and

$$\lim \|T\left((1-u_\lambda)a\right)\| = \lim \|(1-u_\lambda)Ta\| = \|(Ta)^{\cdot}\| = \|b\|$$

(we used here a well known fact [8], Section 3.1.3, that if e_λ is any approximate unit then

$$\lim_\lambda \|x(1-e_\lambda)\| = \|\dot{x}\|,$$

for any $x \in A$). Hence $\inf_{y \in \overline{TI}} \|Ta_0 + y\| \leq \|b\|$. Since the norm of any lift of b is not less than $\|b\|$ we get $\inf_{y \in \overline{TI}} \|Ta_0 + y\| = \|b\|$.

Thus $dist(Ta_0, \overline{TI}) = \inf_{y \in \overline{TI}} \|Ta_0 + y\| = \|b\|$ and, by Theorem 1, there is $y_0 \in \overline{TI}$ such that $\|Ta_0 + y_0\| = \|b\|$. Since $Ta_0 + y_0 \in \overline{E}$, we are done. □

Lemma 3. $\overline{\dot{a}(A/I)\dot{a}} = \overline{aAa}/\overline{aIa}$.

Proof. Define a map $f : aAa \to \dot{a}(A/I)\dot{a}$ by $f(aba) = \dot{a}\dot{b}\dot{a}$ and extend it by continuity to $\tilde{f} : \overline{aAa} \to \overline{\dot{a}(A/I)\dot{a}}$. Since $\tilde{f}|_{\overline{aIa}} = 0$ there is a well-defined map $F : \overline{aAa}/\overline{aIa} \to \overline{\dot{a}(A/I)\dot{a}}$. This is surjection because its image is closed (as the image of any $*$-homomorphism between C^*-algebras) and contains $\dot{a}(A/I)\dot{a}$.

Suppose that $x \in \overline{aAa}/\overline{aIa}$ and $F(x) = 0$. Let $X \in \overline{aAa}$ be any preimage of x. Then there are elements $b_n \in A$ such that $X = \lim ab_n a$. Then $\lim \dot{a}\dot{b_n}\dot{a} = 0$ and there are $i_n \in I$ such that $ab_n a - i_n \to 0$. Hence $X \in I$. Let $\{u_\lambda\}$ be a quasicentral approximate unit for $I \subseteq A$. Then for any ϵ, there is n such that $\|X - ab_n a\| \leq \epsilon$ and there is λ such

that $\|X - Xu_\lambda\| \leq \epsilon$, $\|ab_n au_\lambda - ab_n u_\lambda a\| \leq \epsilon$. Then $\|X - ab_n u_\lambda a\| \leq 3\epsilon$, $dist(X, aIa) \leq 3\epsilon$ whence $X \in \overline{aIa}$. □

A C^*-algebra A is called SAW* ([13]) if for any $a, b \in A_{sa}$, $ab = 0$, there is $0 \leq z \leq 1$, $z \in A^+$, such that $az = a$, $zb = 0$.

Lemma 4. *If A is SAW*, $p \in A$ is a projection, then pAp is SAW*.*

Proof. Suppose $a, b \in (pAp)_{sa}$, $ab = 0$. Since A is SAW* there is $0 \leq z \leq 1$, $z \in A^+$, such that $az = a$, $zb = 0$. Clearly $0 \leq pzp \leq 1$, $pzp \in (pAp)^+$, $apzp = a$, $pzpb = 0$. □

Below we write $d << e$ for selfadjoint elements satisfying $de = ed = d$.

Theorem 5. *([9]) Let A be a C^*-algebra, I — its ideal, such that the quotient A/I is SAW*. Let $b \in A/I$ be a nilpotent of order n. Then there are elements*

$$0 \leq d_{n-1} << e_{n-1} << d_{n-2} << e_{n-2} << \ldots << e_2 << d_1 << e_1 \leq e_0 = 1 \qquad (1)$$

and their lifts

$$0 \leq D_{n-1} << E_{n-1} << D_{n-2} << E_{n-2} << \ldots << E_2 << D_1 << E_1 \leq E_0 = 1 \qquad (2)$$

such that
(i) $b = \sum_{j=1}^{n-1}(e_{j-1} - e_j)bd_j$,
(ii) for any lift B of b, $\sum_{j=1}^{n-1}(E_{j-1} - E_j)BD_j$ is a nilpotent lift of b of order n.

Now let b be an algebraic element of a C^*-algebra with the minimal polynomial $p(t) = (t-\lambda_1)^{k_1} \ldots (t-\lambda_n)^{k_n}$. In [5] a family of n pairwisely orthogonal projections was associated with b in the following way. Let e_i be the spectral idempotents for b and $s = \sum e_i^* e_i$. One can prove that $e_i(b - \lambda_i)^{k_i} e_i = 0$ for each i, s is invertible and

$$p_i = s^{1/2} e_i s^{-1/2} \qquad (3)$$

are projections with sum 1. Let $c = s^{1/2} b s^{-1/2}$. Then

$$p_i(c - \lambda_i)^{k_i} p_i = 0, \qquad (4)$$

$$\sum_{i=1}^{n} p_i c p_i = c. \qquad (5)$$

Theorem 6. *([5]) The following are equivalent:*
(i) $b \in A/I$ has a lift $\tilde{b} \in A$ with $p(\tilde{b}) = 0$;
(ii) the family $\{p_i\}$ can be lifted to a family of projections in A with sum 1.

Since in what follows we use ideas from [5], we now sketch arguments from [5] of how (ii) implies (i). Let a family of projections $\{Q_i\}$ with $\sum Q_i = 1$ be a lift of $\{p_i\}$. Since $p_i(c - \lambda_i)p_i$ is a nilpotent of order k_i in a C^*-algebra $p_i A/I p_i = Q_i AQ_i/Q_i IQ_i$, by [9] it has a nilpotent lift $t_i \in Q_i AQ_i$ of order k_i. Now one can check that $\sum t_i + \lambda_i Q_i$ is a lift of c and $p(\sum t_i + \lambda_i Q_i) = 0$. As a lift of b we can take $S^{-1/2}(\sum t_i + \lambda_i Q_i)S^{1/2}$, where S is any invertible positive lift of s (to find such S we can take any selfadjoint lift of s and then use functional calculus).

2. Main results

Lemma 7. *Let A be a C^*-algebra, I its ideal such that A/I is SAW* and let $b \in A/I$, $p(b) = 0$, $\|b\| = 1$, and a family of projections $\{p_i\}$ associated with b by (3) can be lifted to a family of projections in A with sum 1. Then there is a lift $\tilde{b} \in A$ of b such that $p(\tilde{b}) = 0$, $\|\tilde{b}\| = 1$.*

Proof. Let s, c be as above and projections $\{Q_i\}$ in A be lifts of $\{p_i\}$ with sum 1. By (4), $p_i(c - \lambda_i)p_i$ is a nilpotent of order k_i in the C^*-algebra $p_i(A/I)p_i$. By Lemma 3, $p_i(A/I)p_i = Q_iAQ_i/Q_iIQ_i$. It follows from Lemma 4 and Theorem 5 that there are $e_j^{(i)}, d_j^{(i)} \in p_i(A/I)p_i$ and their lifts $E_j^{(i)}, D_j^{(i)} \in Q_iAQ_i$ satisfying (1) and (2) such that

$$p_i(c - \lambda_i)p_i = \sum_{j=1}^{k_i-1}(e_{j-1}^{(i)} - e_j^{(i)})(c - \lambda_i)d_j^{(i)}$$

and, for any lift C of c,

$$\sum_{j=1}^{k_i-1}(E_{j-1}^{(i)} - E_j^{(i)})(C - \lambda_i)D_j^{(i)}$$

is a nilpotent lift of $p_i(c - \lambda_i)p_i$ of order k_i. Let $q : A \to A/I$ be the canonical surjection. We have

$$q\left(\sum_{i=1}^{n}\sum_{j=1}^{k_i-1}\left((E_{j-1}^{(i)} - E_j^{(i)})(C - \lambda_i)D_j^{(i)} + \lambda_iQ_i\right)\right) =$$

$$\sum_{i=1}^{n}\sum_{j=1}^{k_i-1}\left((e_{j-1}^{(i)} - e_j^{(i)})(c - \lambda_i)d_j^{(i)} + \lambda_ip_i\right) = \sum_{i=1}^{n}p_i(c - \lambda_i)p_i + \lambda_ip_i = \sum_{i=1}^{n}p_icp_i = c.$$

Denote $\sum_{j=1}^{k_i-1}(E_{j-1}^{(i)} - E_j^{(i)})(C - \lambda_i)D_j^{(i)}$ by X_i. Since $X_i^{k_i} = 0$, $X_iQ_j = Q_jX_i = 0$ if $i \neq j$, and $\sum Q_i = 1$ we have

$$p\left(\sum_{i=1}^{n}\sum_{j=1}^{k_i-1}\left((E_{j-1}^{(i)} - E_j^{(i)})(C - \lambda_i)D_j^{(i)} + \lambda_iQ_i\right)\right) =$$

$$\prod_{l=1}^{n}\left(\sum_{i=1}^{n}(X_i + \lambda_iQ_i) - \lambda_l\right)^{k_l} = \prod_{l=1}^{n}\left(\sum_{i=1}^{n}(X_i + (\lambda_i - \lambda_l)Q_i)\right)^{k_l} =$$

$$\prod_{l=1}^{n}\left(X_l + \sum_{i\neq l}(X_i + \lambda_i - \lambda_l)Q_i\right)^{k_l} = \prod_{l=1}^{n}\left(\sum_{i\neq l}(X_i + \lambda_i - \lambda_l)Q_i\right)^{k_l} = 0.$$

It follows from (2) that $\sum_{j=1}^{k_i-1}(E_{j-1}^{(i)} - E_j^{(i)})D_j^{(i)} = Q_i$ whence

$$\sum_{i=1}^{n}\sum_{j=1}^{k_i-1}\left((E_{j-1}^{(i)} - E_j^{(i)})(C - \lambda_i)D_j^{(i)} + \lambda_iQ_i\right) = \sum_{i=1}^{n}\sum_{j=1}^{k_i-1}(E_{j-1}^{(i)} - E_j^{(i)})CD_j^{(i)}.$$

Let $S \in A$ be any invertible positive lift of s. Since $b = s^{-1/2}cs^{1/2}$ the set

$$\mathcal{E} = \left\{\sum_{i=1}^{n}\sum_{j=1}^{k_i-1}S^{-1/2}(E_{j-1}^{(i)} - E_j^{(i)})S^{1/2}XS^{-1/2}D_j^{(i)}S^{1/2} \mid X \in A,\ q(X) = b\right\}$$

consists of lifts of b and we have $p(X) = 0$, for any $X \in \mathcal{E}$. Then $\overline{\mathcal{E}}$ also consists of lifts of b and $p(X) = 0$, for any $X \in \overline{\mathcal{E}}$.

Define an elementary operator $T : A \to A$ by

$$T(X) = \sum_{i=1}^{n} \sum_{j=1}^{k_i - 1} S^{-1/2}(E_{j-1}^{(i)} - E_j^{(i)})S^{1/2} X S^{-1/2} D_j^{(i)} S^{1/2}.$$

Then $\mathcal{E} = \{TX | X \in A,\ q(X) = b\}$. By Corollary 2, in $\overline{\mathcal{E}}$ there is an element of norm 1. □

Theorem 8. *The universal C^*-algebra of a relation $p(b) = 0, \|b\| \leq 1$ is semiprojective.*

Proof. By Theorem 14.1.7 of [6], it suffices to prove that if $b \in M(I)/I$, where $I = \overline{\bigcup I_n}$, $I_1 \subseteq I_2 \subseteq \ldots$, $p(b) = 0$, $\|b\| \leq 1$, then there exists N and $\tilde{b} \in M(I)/I_N$ such that $q_{n,\infty}(\tilde{b}) = b$, $p(\tilde{b}) = 0$, $\|\tilde{b}\| \leq 1$. Clearly we can assume $\|b\| = 1$ and look for $\tilde{b}$ with $\|\tilde{b}\| = 1$.

By [2], there exists N such that $\{p_i\}$ can be lifted to a family $\{Q_i\}$ of projections in $M(I)/I_N$ with sum 1. Denote $M(I)/I$ by A, I/I_N — by J. Then $A/J \cong M(I)/I$. By Kasparov's Technical Theorem, any corona-algebra is SAW*, so the assertion follows now from Lemma 7. □

Recall that a C^*-algebra is residually finite-dimensional (RFD) if the intersection of kernels of its finite-dimensional representations is zero.

Theorem 9. *The universal C^*-algebra of a relation $p(b) = 0, \|b\| \leq 1$ is residually finite-dimensional.*

Proof. Let H be $l^2(\mathbb{N})$. We will identify M_n with $B(l^2\{1, \ldots, n\}) \subseteq B(H)$. Let $\mathcal{A} \subseteq \prod M_n$ be a C^*-algebra of all sequences $\{x_n\}$ such that $*$-strong limit x_n exists and let $\mathcal{I}$ be an ideal of all sequences $\{x_n\}$ such that $*$-strong limit x_n is zero. Then a map $i : \mathcal{A}/\mathcal{I} \to B(H)$ sending each sequence $\{x_n\}$ to its $*$-strong limit is an isomorphism. The algebra $B(H)$ is SAW*. Indeed if $T, S \in B(H)$, $TS = 0$ then for Z equal to the projection on $(RanS)^{\perp}$ one has $TZ = T$, $ZS = 0$. Thus $\mathcal{A}/\mathcal{I}$ is SAW*. Let $p_1, \ldots, p_n$ be a family of projections in $\mathcal{A}/\mathcal{I}$ with sum 1. Let H_j be a range of projection $i(p_j)$, $\{e_i^{(j)}\}_{i \in \mathbb{N}}$ be an orthonormal basis in H_j. Then $e_1^{(1)}, \ldots, e_1^{(n)}, e_2^{(1)}, \ldots, e_2^{(n)}, \ldots$ is an orthonormal basis in H. Let R_k be a projection on the first k basis vectors. Then sequences $\{R_k P_j R_k\}_{k \in \mathbb{N}} \in \mathcal{A}$ are projections with sum 1 and are lifts of P_j. By Lemma 7 each $*$-homomorphism from the universal C^*-algebra of a relation $p(b) = 0, \|b\| \leq 1$ to $\mathcal{A}/\mathcal{I}$ lifts to an $*$-homomorphism to $\mathcal{A}$. Let π be the universal representation of the universal C^*-algebra of a relation $p(b) = 0, \|b\| \leq 1$. Then any lift of $i^{-1} \circ \pi$ has to be injective and hence gives a separating family of finite-dimensional representations. □

Remark 10. *The idea to prove RFD using sequences of matrices having $*$-strong limit has been considered for years. First time it appeared probably in* [4].

The following result is a generalization of Olsen's result ([10]) that each polynomially compact operator is a sum of algebraic and compact.

Theorem 11. *Let $T \in B(H)$, $p(x)$ a polynomial in one variable, and suppose that $p(T)$ is compact. Then there is a compact operator K such that $p(T + K) = 0$, $\|T + K\| = \|T\|_e$.*

Proof. Let b be the image of T in Calkin algebra $B(H)/K(H)$. Then $p(b) = 0$. The assertion follows now from Lemma 7, Kasparov's Technical Theorem and the well-known fact that orthogonal projections (with sum 1) in Calkin algebra can be lifted to orthogonal projections (with sum 1) in $B(H)$. □

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